Evaluation And Comparison Of The Different Standards Used To Define The Positional Accuracy And Repeatability Of Numerically Controlled Machining Center Axes

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Introduction

Machine tool positional accuracy and repeatability are core descriptors of a machine tool and indicate the machine's expected level of performance. While a number of standards and guidelines exist outlining how to evaluate machine tool positional accuracy and repeatability, they differ in their analysis procedures and in key parameter definition. As a result the values reported for positional accuracy and repeatability for any one machine can vary depending on which standard was used. As all standards are equally valid it is beneficial to be aware how the standards differ from each other and how the different calculated values compare to each other. This document aims to do this. The aims of this report can be explicitly broken down into the following:

- List commonly used international or national standards that are related to machine tool positional accuracy and repeatability.
- Determine which standards are equivalent to each other and subsequently isolate the core standards.
- Compare the key parameters from each standard and identify identical and conceptually similar parameters.
- Perform numerical analysis to evaluate how similar parameters compare to each other under different conditions and determine if conversion factors exist allowing parameters from different standards to be directly compared.

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Standards relevant to the study

The core standards included in this study are listed in table 1. These standards primarily deal with determining the positional repeatability and accuracy of a machine tool. Standards specifically focused on the geometric accuracy of the machine tool are not included in the study.

Table 1: Core standards under investigation.

Name	Title	Comment
ISO 230-2:2006	Determination of accuracy and repeatability of positioning numerically controlled machine axes	This replaced the ISO 230-2:1997 version of the standard. Details of the changes are given later.
ISO 230-2:1997	Determination of accuracy and repeatability of positioning numerically controlled machine axes	
ISO 230-2:1988	Determination of accuracy and repeatability of positioning of numerically controlled machine tools	This was replaced by ISO 230-2:1997. Differences between the two standards are given later.
England		
BS ISO 230-2 (1999)	Determination of accuracy and repeatability of positioning numerically controlled machine axes	Equivalent to ISO 230-2:1997
German		
VDI/DGQ 3441	Statistical testing of the operational and positional accuracy of machine tools	
DIN ISO 230-2 (2000)	Determination of accuracy and repeatability of positioning numerically controlled machine axes	Equivalent to ISO 230-2:1997
USA		
ASME B5.54(2005)	Methods for performance evaluation of computer numerically controlled machining centers	Data analysis for machine tool accuracy is equivalent to ISO 230-2:1997
China		
GB/T 1721.2 (2000)	Determination of accuracy and repeatability of positioning numerically controlled machine axes	Equivalent to ISO 230-2:1997
Japan		
JIS B 6192:1999	Determination of accuracy and repeatability of positioning numerically controlled machine axes	Equivalent to ISO 230-2:1997
JIS B 6330-1980	Test code for performance and accuracy of numerically controlled machine tools	This standard is withdrawn however details on the analysis techniques will be given.

Key Standards

The following standards will be looked at in detail;

- ISO 230-2:2006
- ISO 230-2:1997
- ISO 230-2:1988
- VDI/DQG 3441
- ANSI B5:54
- JIS B 6330-1980

While details of each standard are not given, specifics with respect to the history of the standard are provided, i.e. which standard it replaced, which standards replaced it, which standards are based on the standard etc.

NOTE: This document is not a substitute for reading the individual standards. The actual standards should be used when undertaking any of the outlined tests or in determining the positional accuracy of a machine tool.

ISO 230-2: 2006

Test Code for Machine tools - Part 2: Determination of accuracy and repeatability of positioning numerically controlled axes.

Older Versions :

- ISO 230-2:1997
- ISO 230-2:1988

Other international standards based on ISO 230-2:2006 standard - NONE

Scope of the standard:

To specify the methods of testing and evaluating the accuracy and repeatability of positioning of NC machine tools and components by direct measurement of independent axes on the machine. Used for type testing, acceptance testing, comparison testing, periodic verification, machine compensation.

Differences between ISO 230-2:2006 and ISO 230-2:1997:

- A measurement uncertainty statement is added to the 2006 version and now the measurement uncertainty should be included when reporting the key parameters. An annex is provided with the standard detailing how to determine the measurement uncertainty.

ISO 230-2: 1997

Test Code for Machine tools - Part 2: Determination of accuracy and repeatability of positioning numerically controlled axes.

Older Versions :

- ISO 230-2:1988

Other international standards based on the ISO 230-2:1997 standard

- **GB/T 17421.2 (2000)** (China) Test code for machine tools-Part 2: Determination of accuracy and repeatability of positioning numerically controlled axes. Equivalent to ISO 230-2:1997
- **JIS B 6192 (1999)** (Japan) Test code for machine tools-Part 2: Determination of accuracy and repeatability of positioning numerically controlled axes. Equivalent to ISO 230-2:1997
- **BS ISO 230-2 (1999)** (England) Test code for machine tools-Part 2: Determination of accuracy and repeatability of positioning numerically controlled axes. Equivalent to ISO 230-2:1997
- **DIN ISO 230-2 (2000)** (Germany) Test code for machine tools-Part 2: Determination of accuracy and repeatability of positioning numerically controlled axes. Equivalent to ISO 230-2:1997

Scope of the standard:

To specify the methods of testing and evaluating the accuracy and repeatability of positioning of NC machine tools and components by direct measurement of independent axes on the machine. Used for type testing, acceptance testing, comparison testing, periodic verification, machine compensation.

Differences between ISO 230-2:1997 and ISO 230-2:1988:

1. The 1997 version changed some of the terminology. The term 'standard uncertainty' is used instead of 'standard deviation' to avoid making assumptions with respect to the distribution of the measured data.
2. The 1997 version uses an expanded uncertainty coverage factor of two (k=2) instead of three (k=3).
3. Calculation of the bidirectional systematic positional deviation of an axis, E, is added to correlate to the 'Accuracy' term in the ANSI B5.54 (1992).
4. Calculation of the range of the bidirectional positional deviation range, M is added and it is equivalent to the Positional Deviation term, P_a, as described in the VDI/DGQ 3441.

ISO 230-2:1988

Acceptance Code for Machine tools - Part 2: Determination of accuracy and repeatability of positioning numerically controlled machine tools.

Older Versions : NONE

Other international standards based on the ISO 230-2:1988 standard: -NONE

Scope of the standard:
To specify the methods of testing and evaluating the accuracy and repeatability of positioning of NC machine tools and components by direct measurement of independent axes on the machine.

VDI/ DQG 3441

Statistical testing of the operational and positional accuracy of machine tools
Translated from the German issue 3/1977

Older Versions: -NONE

Other international standards based on the VDI/DGQ 3441 standard -NONE

Scope of the guidelines:
The guideline describes how statistical methods can be applied to machines that are and are not tied to a particular part to determine operational or positional accuracy of a machine. The standard is in two sections, one section focuses on operational uncertainty, this is measured by determining how accurately a machine can manufacture a defined workpiece. The second section details how the positional accuracy of the machine can be determined from direct length measurement, i.e. under unloaded conditions. It is this latter section of the guidelines that is referred to in this document.

ASME B5.54-2005

Methods for performance evaluation of computer numerically controlled machining centers

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Older Versions:
ASME B5.54-1992

Other international standards based on ASME B5.54-2005 standard - NONE

Other standards that reference the ASME B5.54-2005 standard - NONE

Differences between ASME B5.54 2005 and ASME B5.54-1992:

1. Changes were made to bring consistency to terminology used between this standard and ASME B5.57 -1998 methods for performance evaluation of computer numerically controlled lathes and turning machines.

Scope of the standard:
This standard is very comprehensive and includes methodologies to specify machine tool geometric parameters, positional accuracy and repeatability. It also includes information with respect to environmental conditions and thermal uncertainties.
The section on positional accuracy and repeatability is very similar in approach to ISO 230-2:2006 however it includes a section on periodic error (short wavelength periodic errors).

JIS B 6330-1980

Test code for performance and accuracy of numerically controlled machine tools

This standard was withdrawn in 1987 According to the JSA webpage it was replaced by JIS B 6201. The JIS B 6201 was first implemented in 1953. This standard has been revised and reaffirmed several times in the past. The last revision was in 1993 and this has been reaffirmed in 1998 and 2002. The JIS B 6201-1993 standard does not explicitly outline tests to measure the machine tool positional accuracy and repeatability, perhaps earlier versions did. The JIS B 6192-1999 which was established in 1999 does however outline how to measure machine tool positional accuracy and the analysis section of the standard is as per 1SO 230-2:1997.

Scope of the standard:
The standard outlines a method for determining the positional accuracy and repeatability of a machine tool

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Key Parameters recommended for reporting by the different standards.

Table 2: Parameters recommended for reporting.

ISO 230-2: 2006	ISO 230-2: 1997	ISO 230-2: 1988	VDI/DQG 3441	ANSI B54.5 2005	JIS B 6330-1980
A_{06} & uncertainty (k=2)	A_{06}	A_{88}	P_a	A_{06}	Positioning accuracy test (P_{jis})
$A\uparrow_{06}$ and $A\downarrow_{06}$	$A\uparrow_{06}$ and $A\downarrow_{06}$	R_{88}	P_{smax}	$A\uparrow_{06}$ and $A\downarrow_{06}$	Repeatability test (R_{jis})
E_{06} & uncertainty (k=2)	E_{06}	$R\uparrow_{88}$ and $R\downarrow_{88}$	$\overline{P}_s$	E_{06}	Lost motion test (U_{jis})
$E\uparrow_{06}$ and $E\downarrow_{06}$	$E\uparrow_{06}$ and $E\downarrow_{06}$	$\overline{B}_{88}$	U_{max}	$E\uparrow_{06}$ and $E\downarrow_{06}$	Least input increment test
M_{06} & uncertainty (k=2)	M_{06}		$\overline{U}$	M_{06}	
R_{06} & uncertainty (k=2)	R_{06}			R_{06}	
$R\uparrow_{06}$ and $R\downarrow_{06}$	$R\uparrow_{06}$ and $R\downarrow_{06}$			$R\uparrow_{06}$ and $R\downarrow_{06}$	
B_{06} & uncertainty (k=2)	B_{06}			B_{06}	
$\overline{B}_{06}$	$\overline{B}_{06}$			$\overline{B}_{06}$	
				P_{06}	

Appendix A gives a full list of the nomenclature for each of the standards.

Note 1: While the same parameter notation is used in ISO 230-2:1988, 1997 and 2006 the actual mathematical equations may vary therefore a two digit subscript (i.e. $_{06}$ or $_{88}$) is used to denote the year of the standard being referred to. The equations and notation used in ISO 230-2:1997 and ASME B54.5 are identical to those used in ISO 230-2:2006 and therefore the $_{06}$ subscript is used when referring to ISO 230-2:1997 and ASME B54.4 parameters.
Note 2: As no official abbreviations are given in the JIS 6330 Standard for the different parameters, names have been assigned in this report, i.e. P_{jis}, U_{jis} and R_{jis}.

Comparison between the ISO 230-2:2006 and the VDI/DQG 3441

Table 3: Identical and similar parameters in the ISO 230-2:2006 and VDI/DQG 3441.

ISO 230-2:2006	VDI/DQG 3441	Identical or similar
Mean bi-directional positional deviation of an axis, M_{06} $M_{06} = \max[\bar{x}_i] - \min[\bar{x}_i]$	Positional deviation, P_a $P_a = \left\lvert \bar{\bar{x}}_{j\max} - \bar{\bar{x}}_{j\min} \right\rvert$	**Identical** - The difference between the maximum and minimum averaged positional deviation over the forward and reverse directions.
Mean Reversal value of an axis $\bar{B}_{06}$ $\bar{B}_{06} = \frac{1}{m}\sum_{i=1}^{m} B_i$ Where; $B_i = \bar{x}_i \uparrow - \bar{x}_i \downarrow$	Mean Reversal Error, $\bar{U}$ $\bar{U} = \frac{1}{m}\sum_{j=1}^{m} U_j$ Where; $U_j = \left\lvert \bar{x}_j \downarrow - \bar{x}_j \uparrow \right\rvert$	**Similar -** Average reversal error. Due to slight differences in the equations the values may vary. This is especially true if the averaged forward and reverse positional errors lines intersect each other as in figure 4.
Reversal value of an axis $B_{06} = \max[\lvert B_i \rvert]$	Max reversal error at a position $U_{\max} = \max[U_j]$	**Identical -** Maximum reversal error.
Bi-directional accuracy of positioning of an axis, A_{06} $A_{06} = \max[\bar{x}_i \uparrow + 2s_i \uparrow ; \bar{x}_i \downarrow + 2s_i \downarrow]$ $- \min[\bar{x}_i \uparrow - 2s_i \uparrow ; \bar{x}_i \downarrow - 2s_i \downarrow]$	Positional Uncertainty, P $P = \left[\bar{\bar{x}}_j + \frac{1}{2}\left(U_j + P_{sj}\right)\right]_{Max}$ $- \left[\bar{\bar{x}}_j - \frac{1}{2}\left(U_j + P_{sj}\right)\right]_{Min}$	**Similar -** Maximum range of values based on mean positional errors, corresponding standard deviations and reversal errors along the axis. As the positional uncertainty, P uses three times the standard deviation in its calculation and bi-directional accuracy, A_{06}, only uses twice the standard deviation, P is expected to be bigger than A.
Unidirectional repeatability of positioning of an axis, $R_{06}\uparrow$ or $R_{06}\downarrow$ $R_{06}\uparrow = \max[4s_i \uparrow]$ $R_{06}\downarrow = \max[4s_i \downarrow]$	Max Positional Scatter, $P_{s\max}$ $P_{s\max} = P_{sj\max}$ $= \max[6\bar{s}_j]$	**Similar -** Indicates the maximum spread of data points that occurred at an individual target position. P_{smax} will always be larger than either $R_{06}\downarrow$ or $R_{06}\uparrow$ as P_{smax} uses three times the standard deviation in its calculation while $R_{06}\downarrow$, only uses twice the standard deviation, Note that P_{smax} is related to the averaged deviation over the forward and reverse directions.
Repeatability of an Axis, R_{06} $R_{06} = \max[R_i]$	**No equivalent parameter**	If B_{06} and U are zero then $2/3P_{smax}$ should be similar to R_{06}. Otherwise it is expected that $\bar{U} + 2/3\,P_{s\max}$ should be similar to R_{06}.

See Appendix A for details on $x_i \uparrow$, $\bar{\bar{x}}_j$, $s_i \uparrow$ and $\bar{s}_j$

Comparison between ISO 230-2:2006 and ISO 230-2:1988

Table 4: Identical and similar parameters in the ISO 230-2:2006 and ISO 230-2:1988.

ISO 230-2:2006	ISO 230-2:1988	Comparison
Mean Reversal value of an axis $\overline{B}_{06}$ $\overline{B} = \frac{1}{m}\sum_{i=1}^{m} B_i$	Mean Reversal value of an axis $\overline{B}_{88}$ $\overline{B} = \frac{1}{n}\sum_{j=1}^{n} B_j$	**Identical** - Averaged reversal error
Bi-directional accuracy of positioning of an axis, A_{06} $A_{06} = \max\left[\bar{x}_i \uparrow +2s_i \uparrow ; \bar{x}_i \downarrow +2s_i \downarrow\right] - \min\left[\bar{x}_i \uparrow -2s_i \uparrow ; \bar{x}_i \downarrow -2s_i \downarrow\right]$	Bi-directional accuracy of positioning of an axis, A_{88} $A_{88} = \max\left[\bar{x}_j \uparrow +3s_j \uparrow ; \bar{x}_j \downarrow +3s_j \downarrow\right] - \min\left[\bar{x}_j \uparrow -3s_j \uparrow ; \bar{x}_j \downarrow -3s_j \downarrow\right]$	**Similar -** but ISO 230-2:1988 uses three standard deviations in its calculations while the 2006 version uses two standard deviations. A_{88} is expected to be greater than A_{06}
Unidirectional repeatability of positioning of an axis, $R\uparrow_{06}$ or $R\downarrow_{06}$ $R\uparrow_{06} = \max\left[4s_i \uparrow\right]$ $R\downarrow_{06} = \max\left[4s_i \downarrow\right]$	Unidirectional repeatability of positioning of an axis, $R\uparrow_{88}$ or $R\downarrow_{88}$ $R\uparrow_{88} = \max\left[6s_j \uparrow\right]$ $R\downarrow_{88} = \max\left[6s_j \downarrow\right]$	**Similar-** direct conversions between the two standards exist, $R\uparrow_{06} = 2/3\ R\uparrow_{88}$ $R\downarrow_{06} = 2/3\ R\downarrow_{88}$ Likewise $R\uparrow_{88} = 3/2\ R\uparrow_{06}$ $R\downarrow_{88} = 3/2\ R\downarrow_{06}$
Repeatability of positioning of an axis, R_{06} $R_{06} = \max\left[R_i\right]$ Where; $R_i = \max\left[2s_i \uparrow +2s_i \downarrow +\lvert B_i \rvert ; R_i \uparrow ; R_i \downarrow\right]$	Repeatability of positioning of an axis, R_{88} $R_{88} = \max\left[R_j\right]$ Where; $R_j = \max\left[3s_j \uparrow +3s_j \downarrow +\lvert B_j \rvert ; R_j \uparrow ; R_j \downarrow\right]$	**Similar-** Based on same concept, however ISO 230-2:1988 uses three standard deviations in its calculation as opposed to two standard deviations used in ISO 230-2:2006. R_{88} will be bigger than R_{06}.

See Appendix A for details on $x_i \uparrow$, $x_j \uparrow$, $s_i \uparrow$ and $s_j \uparrow$

Comparison between ISO 230-2:1988 and VDI/DQG 3441

Table 5: Identical and similar parameters in the ISO 230-2:1988 and VDI/DQG 3441.

ISO 230-2:1988	VDI/DQG 3441	Identical or similar
Mean Reversal value of an axis $\overline{B}_{88}$ $\overline{B}_{88} = \frac{1}{n}\sum_{j=1}^{n} B_j$ Where; $B_j = \bar{x}_j \uparrow - \bar{x}_j \downarrow$	Mean Reversal Error, $\overline{U}$ $\overline{U} = \frac{1}{m}\sum_{j=1}^{m} U_j$ Where; $U_j = \left\lvert \bar{x}_j \downarrow - \bar{x}_j \uparrow \right\rvert$	**Similar -** Average reversal error. Due to slight differences in the equations the values may vary. This is especially true if the averaged forward and reverse positional errors lines intersect each other as in figure 4.
Bi-directional accuracy of positioning of an axis, A_{88} $A_{88} = \max\left[\bar{x}_j \uparrow + 3s_j \uparrow ; \bar{x}_j \downarrow + 3s_j \downarrow\right]$ $- \min\left[\bar{x}_j \uparrow - 3s_j \uparrow ; \bar{x}_j \downarrow - 3s_j \downarrow\right]$	Positional Uncertainty, P $P = \left[\bar{\bar{x}}_j + \frac{1}{2}\left(U_j + P_{sj}\right)\right]_{Max}$ $- \left[\bar{\bar{x}}_j - \frac{1}{2}\left(U_j + P_{sj}\right)\right]_{Min}$	**Similar -** Maximum range of values based on mean positional error and corresponding standard deviations about each target position. As the positional uncertainty, P uses the averaged standard deviation over the forward and reverse directions it is expected to be slightly smaller than the bi-directional accuracy , A_{88}.
Unidirectional repeatability of positioning of an axis, $R_{88}\uparrow$ or $R_{88}\downarrow$ $R_{88}\uparrow = \max\left[6s_j \uparrow\right]$ $R_{88}\downarrow = \max\left[6s_j \downarrow\right]$	Max Positional Scatter, $P_{s\max}$ $P_{s\max} = P_{sj\max}$ $= \max[6\bar{s}_j]$	**Similar -** Indicates the maximum spread of data points that occurred at an individual target position. As P_{smax} is based on the averaged standard deviation it is expected to be slightly smaller than $R_{88}\uparrow$ or $R_{88}\downarrow$
Repeatability of positioning of an axis, R_{88} $R_{88} = \max\left[R_j\right]$ Where; $R_j = \max\left[3s_j \uparrow + 3s_j \downarrow + \left\lvert B_j \right\rvert ; R_j \uparrow ; R_j \downarrow\right]$	**No equivalent parameter**	If B_{88} and U are zero then P_{smax} should be similar to R_{88}. Otherwise it is expected that $\overline{U} + P_{s\max}$ should be similar to R_{88}

See Appendix A for details on $x_j \uparrow$, $\bar{\bar{x}}_j$, $s_j \uparrow$ and $\bar{s}_j$

Comparison between VDI/DQG 3441 and JIS B 6330-1980

Table 6: Identical and similar parameters in the VDI/DQG 3441 and JIS 6330-1980.

VDI/DQG 3441	JIS B 6330-1980	Identical or similar
Max reversal error at a position $U_{\max} = \max[U_j]$	Lost Motion test $U_{JIS} = \max\lfloor U_j \rfloor$	**Identical,** however U_{jis} is averaged over 7 measurements at each point and not 5 as per the VDI/DQG
Max Positional Scatter, $P_{s\max}$ $P_{s\max} = P_{sj\max}$ $= \max[6\bar{s}_j]$	Repeatability test, R_{jis} $R_{JIS} = \pm\frac{1}{2}\max[R_{JISi}]$ Where; $R_{JISi} = \max[x_{ij}] - \min[x_{ij}]$	**Similar:** P_{smax} is 6 times the largest averaged standard deviation of data points at a target point, while R_{jis} is ½ the maximum range of data points measured at a target point. R_{jis} will be much smaller than P_{smax}.

Note the number of required measurement points for the JIS B6330 is less than that required for the VDI/DQG 3441.

Comparison between ISO 230-2:2006 and JIS B 6330-1980

Table 7: Identical and similar parameters in the ISO 230-2:2006 and JIS B 6330-1980.

ISO 230-2:2006	JIS B 6330-1980	
Reversal value of an axis $B_{06} = \max[\lfloor B_i \rfloor]$	Lost Motion test $U_{JIS} = \max\lfloor U_j \rfloor$	**Identical,** however U_{jis} is averaged over 7 measurements at each target point and not 5 as per the ISO
Unidirectional repeatability of positioning of an axis, $R_{06}\uparrow$ or $R_{06}\downarrow$ $R_{06}\uparrow = 4s_i\uparrow$ $R_{06}\downarrow = 4s_i\downarrow$	Repeatability test, R_{jis} $R_{JIS} = \pm\frac{1}{2}\max[R_{JISi}]$ Where; $R_{JISi} = \max[x_{ij}] - \min[x_{ij}]$	**Similar:** $R_{06}\uparrow$ is 4 times the largest standard deviation of data points at a target point, while R_{jis} is ½ the maximum range of data points measured at a target point. R_{jis} will be much smaller than $R_{06}\uparrow$.
Bi-directional systematic positional deviation of an axis, E_{06} $E_{06} = \max[\bar{x}_i\uparrow; \bar{x}_i\downarrow]$ $- \min[\bar{x}_i\uparrow; \bar{x}_i\downarrow]$	Positional accuracy $P_{JIS} = \max\lfloor x_{ij} \rfloor - \min\lfloor x_{ij} \rfloor$	**Similar:** Both terms are similar in concept, but as there are significant differences between the standards regarding the number of data points required the two parameters may vary substantially.

Note the number of required measurement points for the JIS B6330 is less than that required for the ISO 230-2:2006.

Numerical Evaluation

Identical parameters from the different standards need no further explanation. Likewise for parameters that have no comparable parameter in other standards. However it is worth examining the relationship between similar parameters and determining if guidelines can be written that will allow for translation between the conceptually similar parameters defined by the different standards. For example, can the ISO 230-2:2006 bidirectional accuracy of an axis, A, be converted directly in the VDI/DQG 3441 positional uncertainty, P? Numerical analysis was undertaken to determine the ratios between the conceptually similar parameters and the expected range of ±percentage errors associated with each ratio.

Methodology

A Monte Carlo approach is taken whereby several hundred sets of 'measurement points' are generated using the Gaussian random numbers generator function in Matlab. Two different approaches were taken when generating the 'measurement points'. In both cases the generated 'measurement points' were analyzed as per the various standards and the core parameters compared.

Note on Gaussian assumption: In general machine tools are not Gaussian in nature. Most errors are systematic. The first approach taken (details to follow) assumes a purely Gaussian distribution of the 'measurement points', this is somewhat limited in its validity. The second approach (again details follow) while still using Gaussian distributions has a taken some, but not all, of the expected systematic errors into consideration. Neither approach can model machine tools accurately and it should be realized that definitive (100% accurate) conversions between standards are not possible. That said an appreciation of how the different parameters compare to each other is beneficial. It is also worth observing how the starting model assumptions affect the magnitude of the expected ratios.

1. Gaussian Random Numbers

For a chosen set of eleven target positions (relative position of each target point to each other is not important) ten data points (five in the forward direction and five in the reverse) were randomly generated in Matlab using the Gaussian random number generator '`NORMRND`'. This function requires a standard deviation and mean value. The figure 1 below details a typical set of data points created when the mean and standard deviation were taken to be 0.5μm and 5 μm respectively.

Analysis:

Using the same mean and standard deviation five thousand sets of 'measurement points' were generated.. For each of the five thousand sets the VDI/DQG 3441, ISO 230-2:1988, ISO 230-2:2006 and JIS6330 parameters were calculated. While a standard deviation value of 5μm maybe considered somewhat on the large side, it is worth noting that altering the standard deviation used in the model does not significantly affect the resulting ratios or their respective standard deviations.

The ratios between several comparable parameters are listed in the first column of table 10. The values listed are the ratios averaged over five thousand runs while the number in parenthesis is the standard deviation over the five thousand runs.

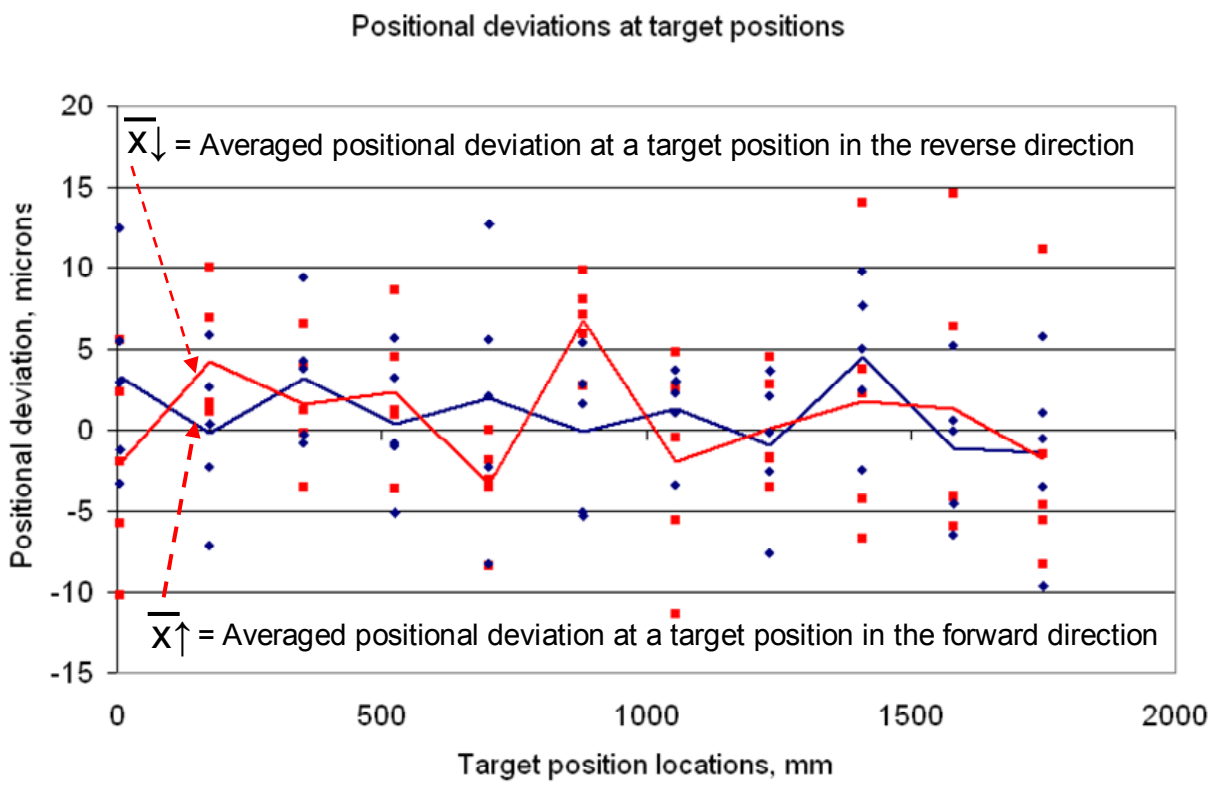

Figure 1: Typical data set generated with a standard deviation of 5µm and a mean value of 0.5µm.

2. Gaussian Random Numbers with Additional Constraints

This approach also uses the Matlab '`NORMRND`' Gaussian random number generator to generate artificial 'measurement points', however there are more constraints with respect to the relative location of the data points to the adjacent target positions and the 'measurement points' in the reverse direction. Figure 3 illustrates a typical set of generated 'measurement points'. The methodology can be described in four steps, please also refer to figure 2. The constraints used are based on ISO 230-2:2006 parameters, see table 8 for descriptions of the parameters used and figure 3 for graphical illustration.

1. $\bar{x}_i \uparrow$, the first target position is randomly selected within a predefined range.
2. The mean value of positional error, $\bar{x}_i$ at each of the subsequent target locations in the forward directions is randomly generated by a Gaussian function with the mean value taken as the position of the target point generated in step 1 and the deviation taken as a typical unidirectional systematic positional divided by four, $E\downarrow/4$ or $E\uparrow/4$. E↑ is the range in which the averaged target position error lies and thus it deviation can be taken as E↑/4 (k=2).
3. Gaussian functions were used to generate the location of the mean positional deviation values at each target position in the reverse direction. The value is determined by the subtracting a number generated using the mean reversal value, $\overline{B}$, as the mean and the

$\overline{B}$ /24 as the standard deviation. Taking this approach the mean positional errors in the forward and reverse directions will not intersect. This issue is addressed in section 2.1

4. Five data points are randomly created for each of the mean positional deviations in the forward and reverse directions. For the Gaussian function the mean value is taken to be the relevant $\overline{x}_i\uparrow$ or $\overline{x}_i\downarrow$ value and the deviation is a typical $R\uparrow/4$ value.

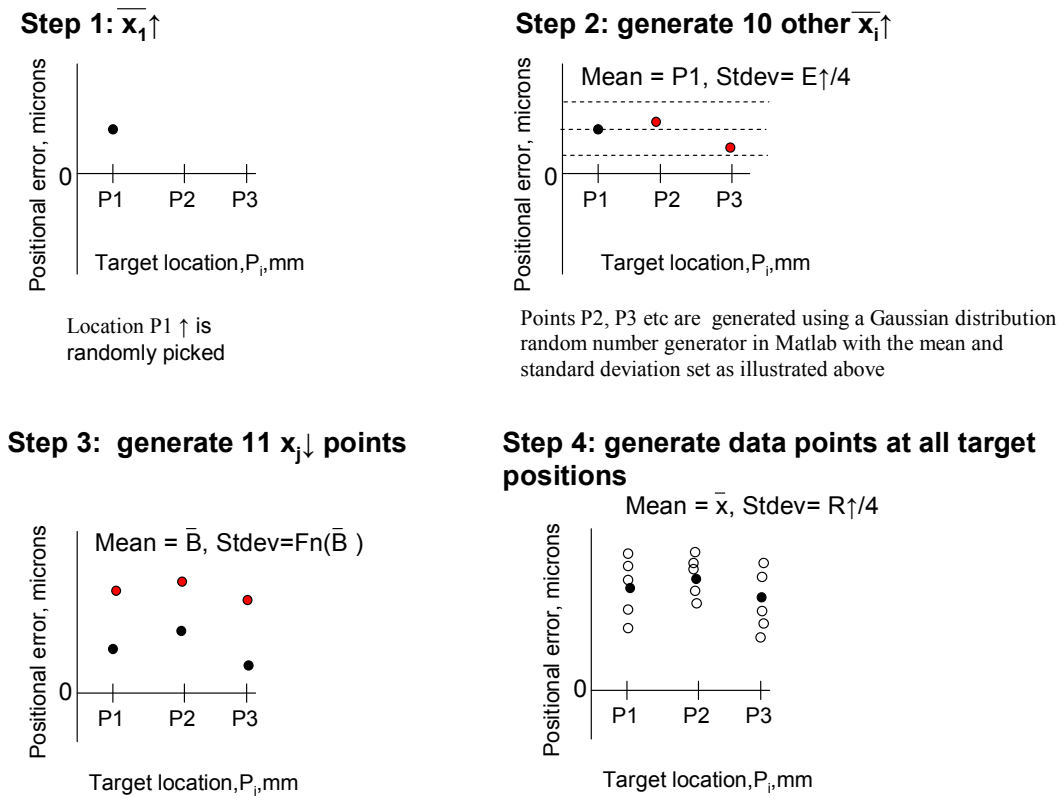

Figure 2: Steps involved in generating the random numbers.

Table 8: Parameters used in generating the "measurement point" sets and their meaning.

Parameter	Meaning
E↑	It gives information with respect to the range of the averaged positional deviations ($\overline{x}_i\uparrow$ or $\overline{x}_i\downarrow$) calculated along the axis in either the forward or the reverse direction. It contains no information with respect to the spread of data points at each measurement position. See table 7 for its numerical definition.
$\overline{B}$	B_i, the reversal value at a target position, P_i, is the difference between the averaged positional deviation in the forward $\overline{x}_i\uparrow$ and the reverse $\overline{x}_i\downarrow$ directions. The mean reversal value recorded along the axis is taken as the average of all the reversal values, B_i, along an axis. See table 3 for its numerical definition.
R↑	R↑, the unidirectional repeatability of positioning at a position in the forward direction, is related to the maximum spread of measured positional errors at a target position, P_i, the spread is taken as $4s_i\uparrow$ (coverage k=2), where s_i is the standard deviation of the measurement points at a target position.

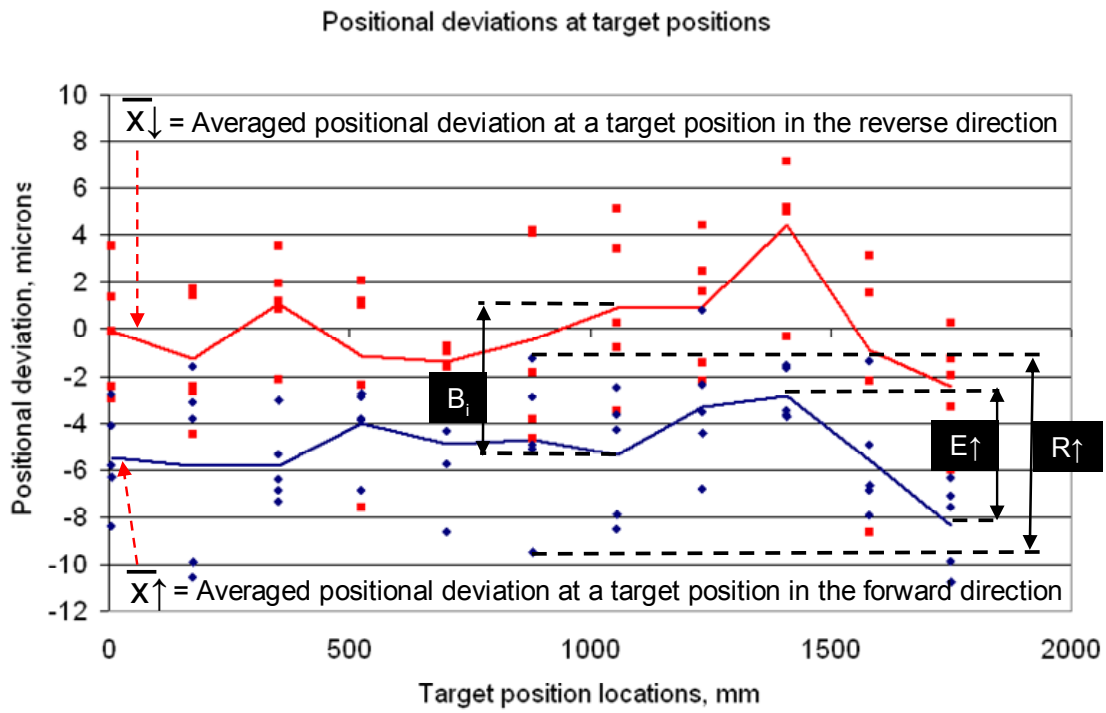

Figure 3: A typical set of data points generated by the second method.

Analysis:

For this approach there are effectively three test parameters ($E\uparrow, \overline{B}, R\uparrow$). Three test sets were run whereby upper and lower limits were given to each of the three parameters. The three testing bands were: 0.1 µm to 5 µm, 5 µm to 10 µm and lastly 10 µm to 30 µm. Within each test set the values were systematically varied (eight different combinations) and five hundred simulations run for each combination, see table 9.

Table 9: Combinations tested for the first bandwidth, 0.1µm to 5µm.

	Run 1	**Run 2**	**Run 3**	**Run 4**	**Run 5**	**Run 6**	**Run 7**	**Run 8**
$E\uparrow$	5	5	5	5	0.5	0.5	0.5	0.5
$\overline{B}$	5	5	0.5	0.5	5	5	0.5	0.5
$R\uparrow$	5	0.5	0.5	5	5	0.5	0.5	5

The key ISO 230-2:1988, ISO 230-2:2006, VDI/DQG 3441 and JIS 6330 parameters were calculated and the ratio between conceptually similar parameters as outlined in tables 3 to 7 were determined. The averaged ratio and the averaged standard deviation (in parenthesis) within each of the three test sets are listed in the last three columns of table 10.

Table 10: Summary of the parameter ratios (and standard deviations) calculated from the Monte Carlo simulations.

Ratio	Gaussian	0.1 μm → 5μm	5 μm → 10μm	10 μm → 30μm
A_{06}/P	0.8 (0.06)	0.9 (0.03)	0.86 (0.05)	0.87 (0.05)
A_{88}/A_{06}	1.45 (0.04)	1.19 (0.03)	1.22 (0.04)	1.22 (0.04)
$A_{88}\uparrow/A_{06}\uparrow$	1.45 (0.04)	1.29 (0.05)	1.33 (0.06)	1.32 (0.06)
A_{88}/P	1.16 (0.1)	1.06 (0.05)	1.05 (0.06)	1.05 (0.06)
$R_{06}\uparrow/P_{smax}$	0.75 (0.1)	0.76 (0.1)	0.76 (0.1)	0.76 (0.1)
$R_{88}\uparrow/P_{smax}$	1.13 (0.15)	1.14 (0.16)	1.14 (0.16)	1.14 (0.16)
$R_{06}/(U+2/3P_{smax})$	1.14 (0.11)	1.06 (0.06)	1.03 (0.05)	1.04 (0.05)
$R_{88}/(U+P_{smax})$	1.16 (0.11)	1.06 (0.06)	1.03 (0.05)	1.03 (0.05)
R_{88}/R_{06}	1.48 (0.03)	1.26 (0.02)	1.27 (0.06)	1.28 (0.06)
$Rjis/R_{06}\uparrow$	0.35 (0.05)	0.35 (0.05)	0.35 (0.05)	0.35 (0.05)
$Rjis/R_{88}\uparrow$	0.26 (0.03)	0.26 (0.03)	0.26 (0.03)	0.26 (0.03)
$Rjis/P_{smax}$	0.23 (0.03)	0.23 (0.03)	0.23 (0.03)	0.23 (0.03)
Pjis/E	0.66 (0.21)	0.73 (0.13)	0.76 (0.12)	0.75 (0.13)

See Notes 1& 2 on page 12 and table 11.

Table 11: Ratio Key.

Ratio	Detail
A_{06}/P	Bi directional Accuracy (ISO 230-2:2006) / Position Uncertainty (VDI/DQG 3441)
A_{88}/A_{06}	Bi directional Accuracy (ISO 230-2:1988) / Bi directional Accuracy (ISO 230-2:2006)
$A_{88}\uparrow/A_{06}\uparrow$	Unidirectional Accuracy (ISO 230-2:1988) / Unidirectional Accuracy (ISO 230-2:2006)
A_{88}/P	Bi directional Accuracy (ISO 230-2:1988) / Position Uncertainty (VDI/DQG 3441)
$R_{06}\uparrow/P_{smax}$	Unidirectional Repeatability (ISO 230-2:2006)/ Positional Scatter (VDI/DQG 3441)
$R_{88}\uparrow/P_{smax}$	Unidirectional Repeatability (ISO 230-2:1988)/ Positional Scatter (VDI/DQG 3441)
$R_{06}/(U+2/3P_{smax})$	Unidirectional Repeatability (ISO 230-2:2006)/ (Reversal error + ⅔Positional Scatter (VDI/DQG 3441))
$R_{88}/(U+P_{smax})$	Unidirectional Repeatability (ISO 230-2:1988)/ (Reversal error + Positional Scatter (VDI/DQG 3441))
R_{88}/R_{06}	Repeatability (ISO 230-2:1988) / Repeatability (ISO 230-2:2006)
$R_{jis}/R_{06}\uparrow$	Repeatability (JIS B 6330-1980)/ Unidirectional Repeatability (ISO 230-2:2006)
$R_{jis}/R_{88}\uparrow$	Repeatability (JIS B 6330-1980)/ Unidirectional Repeatability (ISO 230-2:1988)
R_{jis}/P_{smax}	Repeatability (JIS B 6330-1980)/ Positional Scatter (VDI/DQG 3441)
P_{jis}/E	Positional Accuarcy (JIS B 6330-1980)/ Bi-directional Positional Deviation (ISO 230-2:2006)

2.1. Intersecting ↑and ↓

A limitation of the previous method used to generated 'measurement points' is that the forward and reverse directions will not intersect, a phenomena that may well occur in reality. The program was modified so that the forward and reverse directions were forced to intersect. This was achieved by removing steps 2 and 3 outlined in section 2 and specifying the location of the averaged positional error in each direction, see figure 4 for an example of a typical output. R↑ and E↑ values were used to determine the spread of the data points at a target position and the range of positional errors in either the forward or reverse directions. The program was run 500

times and the results are presented in Table 12. Again the averaged ratio and the averaged standard deviation (in parenthesis) are given for the three test bands.

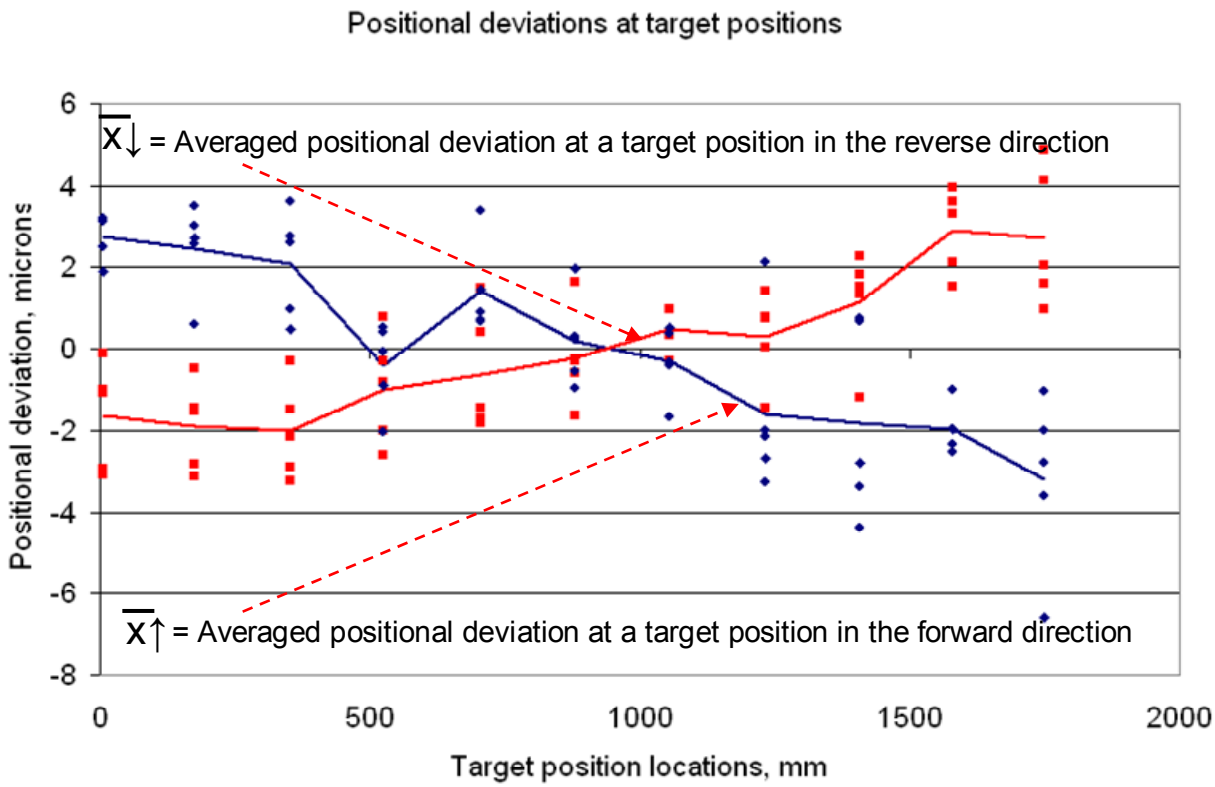

Figure 4: Randomly generated data points where ↑and ↓ are forced to intersect.

Table 12: Summary of the parameter ratios (and standard deviations) calculated from intersecting ↑ and ↓.

Ratio	**0.1 μm → 5μm**	**5 μm → 10μm**	**10 μm → 30μm**
A_{06}/P	0.85 (0.04)	0.82 (0.05)	0.84 (0.05)
A_{88}/A_{06}	1.27 (0.03)	1.29 (0.04)	1.29 (0.04)
$A_{88}\uparrow/A_{06}\uparrow$	1.27 (0.04)	1.29 (0.04)	1.29 (0.04)
A_{88}/P	1.08 (0.06)	1.07 (0.07)	1.08 (0.07)
$R_{06}\uparrow/P_{smax}$	0.76 (0.1)	0.76 (0.1)	0.76 (0.11)
$R_{88}\uparrow/P_{smax}$	1.14 (0.16)	1.14 (0.15)	1.14 (0.16)
$R_{06}/(U+2/3P_{smax})$	1.26 (0.09)	1.17 (0.1)	1.19 (0.1)
R_{88} $(U+P_{smax})$	1.23 (0.09)	1.12 (0.09)	1.08 (0.07)
R_{88}/R_{06}	1.28 (0.04)	1.30 (0.05)	1.29 (0.05)
$R_{jis}/R_{06}\uparrow$	0.34 (0.05)	0.35 (0.05)	0.35 (0.05)
$R_{jis}/R_{88}\uparrow$	0.26 (0.03)	0.26 (0.03)	0.26 (0.03)
R_{jis}/P_{smax}	0.23 (0.03)	0.23 (0.03)	0.23 (0.03)

See Notes 1& 2 on page 12 and table 11.

To provide the reader with a higher degree of transparency, table 13 details the possible percentage variations in the ratio value associated with ± three standard deviations (k=3). In all cases there are significant percentage variations possible. And in certain cases, such as comparing JIS 6330 parameters to ISO 230-2 or VDI/DQG 3441 parameters, or comparing ISO 230-2 repeatability values to VDI/DQG 3441 positional scatter values, the percentage variations associated with the ratios are very large, i.e. > ±34%. In these cases attempting a direction conversion from one standard to another with any degree of certainty is not advised.

Table 13: Ratios and their percentage variations.

Ratio	**Gaussian Ratio +/- % error**	**0.1µm → 5µm Ratio +/- % error**	**Intersecting 0.1µm → 5µm Ratio +/- % error**
A_{06}/P	0.8 ±14.1%	0.9 ±10.0%	0.85 ±14.1%
A_{88}/A_{06}	1.45 ±8.3%	1.19 ±7.6%	1.27 ±7.1%
$A_{88}\uparrow/A_{06}\uparrow$	1.45 ±8.3%	1.29 ±11.6%	1.27 ±9.4%
A_{88}/P	1.16±25.9%	1.06 ±14.2%	1.08 ±16.7%
$R_{06}\uparrow/P_{smax}$	0.75 ±40.0%	0.76 ±39.5%	0.76 ±39.5%
$R_{88}\uparrow/P_{smax}$	1.13 ±39.8%	1.14 ±42.1%	1.14 ±42.9%
$R_{06}/(U+2/3P_{smax})$	1.14 ±28.9%	1.06 ±17.0%	1.26 ±21.4%
$R_{88}/(U+P_{smax})$	1.16 ±28.4%	1.06 ±17.0%	1.23 ±22%
R_{88}/R_{06}	1.48 ±6.1%	1.26 ±4.8%	1.28 ±9.4%
$R_{jis}/R_{06}\uparrow$	0.35 ±42.9%	0.35 ±42.9%	0.34 ±44.1%
$R_{jis}/R_{88}\uparrow$	0.26 ±34.6%	0.26 ±34.6%	0.26 ±34.6%
R_{jis}/P_{smax}	0.23 ±39.1%	0.26 ±39.1%	0.23 ±39.1%
P_{jis}/E	0.66 ±95%	0.73 ±53.4%	

See Note 1 on page 12 and table 11.

Verification of the Monte Carlo predicted ratios.

To verify the Monte Carlo analysis ratios two sets of data points were considered. The first set of points taken were from the worked example in the ISO 230-2:2006, the second set of measurement data points were taken from an actual test carried out on the Monarch milling machine at UNC Charlotte. Both sets of data points underwent VDI/DQG 3441, ISO 230-2:2006 and ISO 230-2:1988 analysis to calculate the key parameters defined by each standard. The ratios between the conceptually similar parameters were determined and compared to the range of ratio values obtained from the Monte Carlo analysis. The results are presented in table 14. The 'Predicted Ratio Ranges' reported in the final column of table 14 are the highest and lowest possible ratios as given by method 2 in the 0.1µm to 5µm test band (based on k=3). 'ISO ratios' in column 2 refers to the ratios obtained based on the data points listed in ISO 230-2:2006, similarly the 'Monarch ratios' were determined from the data points taken off the Monarch machine at UNC Charlotte.

Table 14: Actual ratios compared to predicted ratios.

Ratio	ISO Ratios	Monarch Ratios	Predicted Ratio Ranges
A_{06}/P	0.9	0.82	0.73 → 0.97
A_{88}/A_{06}	1.10	1.23	1.10 → 1.36
A_{88}/P	0.99	1.02	0.91 → 1.26
$R_{06}\uparrow/P_{smax}$	0.75	0.63	0.46 → 1.06
$R_{88}\uparrow/P_{smax}$	1.12	0.95	0.66 → 1.62
$R_{06}/(U+2/3P_{smax})$	0.97	1.18	0.88 → 1.53
$R_{88}/(U+P_{smax})$	0.97	1.13	0.88 → 1.50
R_{88}/R_{06}	1.20	1.32	1.16 → 1.4

See Notes 1 & 2 on page 12 and table 11.

Conclusions

Analysis of the different standards isolated parameters that were identical to each other and those that were similar in concept but mathematically different, see tables 3 to 7. A Monte Carlo approach was taken to determine the relationship between conceptually similar parameters. Two different approaches were taken; purely Gaussian and Gaussian combined with some systematic errors. Twelve relationship pairs were considered, however as the percentage variation associated with the ratios was quite large in some cases (over 40%) only three conversions ratios could be considered (A_{88}/P, $A_{88}\uparrow/A_{06}\uparrow$ and R_{88}/R_{06}). These ratio values still had associated ± percentage variations up 14% and thus great caution should be taken if considering an attempt to converting from one standard to another. Ultimately when seeking to compare values obtained from the different standards there is no substitute for actually conducting the analysis on actual measured data.

Appendix A

Nomenclature used by the various standards

ISO 230-2:2006 & 1997

A↑ & A↓	Unidirectional Accuracy of positioning of an Axis
A	Bidirectional Accuracy of positioning of an Axis
E↑ & E↓	Unidirectional systematic positional deviation of and axis
E	Bidirectional systematic positional deviation of an axis
M	Mean bidirectional positional deviation of an axis, M
B	Reversal value of an axis
R↑ & R↓	Unidirectional repeatability of positioning
R	Bi-directional repeatability of positioning of an axis
$\bar{x}_i \downarrow$ & $\bar{x}_i \uparrow$	Mean unidirectional positional deviation at a target position
$\bar{x}_i$	Mean bidirectional positional deviation at a position
$s_i \downarrow$ & $s_i \uparrow$	Estimator for the unidirectional axis repeatability of positioning at a target point

ISO 230-2:1988

A	Accuracy of an axis
W↑ and W↓	Range of positional deviation
$\bar{B}$	Mean reversal error
B	Reversal value of an axis
R↑ & R↓	Unidirectional repeatability of positioning
R	Bi-directional repeatability of positioning of an axis
$\bar{x}_j \downarrow$ & $\bar{x}_j \uparrow$	Mean unidirectional positional deviation at a target position
$s_j \downarrow$ & $s_j \uparrow$	Estimator for the unidirectional axis repeatability of positioning at a target point

VDI/DQG 3441

P_{smax}	Maximum positional scatter
U_{max}	Maximum reversal error at a position
$\bar{U}$	Mean reversal error
P_a	Positional deviation
P	Positional uncertainty
$\bar{x}_j \downarrow$ & $\bar{x}_j \uparrow$	Mean value of individual values at a position
$\bar{\bar{x}}_j$	System Deviation from the desired value at a target position
$s_j \downarrow$ & $s_j \uparrow$	Standard deviation at a target position in one direction
$\bar{s}_j$	Mean standard deviation at a target position

ASME B5.54 - 2005

A↑ & A↓	Unidirectional Accuracy of positioning of an Axis
A	Bidirectional Accuracy of positioning of an Axis
E↑ & E↓	Unidirectional systematic positional deviation of and axis
E	Bidirectional systematic positional deviation of an axis
M	Mean bidirectional positional deviation of an axis, M
B	Reversal value of an axis
R↑ & R↓	Unidirectional repeatability of positioning
R	Bi-directional repeatability of positioning of an axis
P	Periodic errors
$\bar{x}_i \downarrow$ & $\bar{x}_i \uparrow$	Mean unidirectional positional deviation at a target position
$\bar{x}_i$	Mean bidirectional positional deviation at a position
$s_i \downarrow$ & $s_i \uparrow$	Estimator for the unidirectional axis repeatability of positioning at a target point

JIS 6330-1980

P_{jis}	Positioning Accuracy test
R_{jis}	Repeatability test
U_{jis}	Lost motion test
L_{jis}	Least input increment-feed